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METAPHORIC PERCEPTIONS OF GEOMETRY AMONG PRE-SERVICE PRIMARY SCHOOL TEACHERS

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ABSTRACT

Geometry is one of the most basic areas of mathematics, and it forms the basis for logical reasoning and representation. For pre-service primary school teachers, geometry knowledge is not only a matter of learning definitions and theorems but also the development of effective conceptual frameworks. One of the new trends in mathematics education research is the study of metaphoric perceptions, or how learners conceptualize abstract mathematical notions through metaphorical thinking. This paper reports a theoretical study of the metaphoric perceptions of geometry by pre-service primary school teachers. In addition to the role of metaphor in cognitive and educational development, the paper also provides rigorous geometric definitions, axioms, theorems, proofs, and problem-solving examples to enhance the theoretical framework. This research proposes that metaphors act as a bridge between intuitive spatial experiences and geometric reasoning, influencing the attitudes, understanding, and teaching of future teachers towards geometry.

Keywords: *Geometry education, metaphoric perception, pre-service teachers, Euclidean geometry, spatial reasoning, mathematical cognition.*

1. INTRODUCTION

Geometry is derived from the Greek terms geo, which means "earth," and metron, which means "measurement." Its roots can be found in the practical needs of ancient civilizations. Geometric concepts were established by ancient societies like the Egyptians and Babylonians to measure land, build structures, plan cities, travel across geographical areas, and study celestial patterns. With the help of Greek mathematicians like Euclid, who arranged geometric knowledge into logical systems based on definitions, axioms, and theorems, these practical applications eventually led to the formalization of geometry as an organized subject of study.

The study of space, shapes, sizes, positions, and the relationships between various figures is the focus of geometry, which has developed over time from basic measurement methods into a rigorous and abstract field. These days, it includes a number of subfields, including coordinate geometry, plane geometry, solid geometry, and transformational geometry, each of which offers a different viewpoint on spatial structures. In addition to recognizing shapes, geometry also involves property analysis, relationship proof, and the development of logical reasoning via methodical thought.

Geometry is used as a theoretical and practical tool in the modern world. In addition to supporting sophisticated mathematical ideas utilized in disciplines like physics, computer graphics, robotics, and data visualization, it aids people in comprehending and interpreting the physical world—such as in architecture, engineering, art, and design. Additionally, geometry is essential to education because it fosters critical thinking, spatial vision, and problem-solving abilities. It is therefore crucial to comprehending both our surroundings and the more profound structures of mathematics since it serves as a link between actual real-world experiences and abstract mathematical reasoning.

The study of geometry centers on the basic objects of space, including points, lines, planes, angles, and solids. These basic objects of geometry can be expressed symbolically. For example, a point in the Euclidean plane can be described by an ordered pair:

$$P(x, y)$$

while the general equation of a straight line may be expressed as:

$$y = mx + c$$

where $m$denotes the slope and $c$represents the intercept. Similarly, the equation of a circle, one of the most essential geometric figures, is given by:

$$x^2 + y^2 = r^2$$

where $r$is the radius. Such representations demonstrate that geometry is not only visual but also deeply analytical, combining spatial intuition with algebraic and logical reasoning.

Geometry is a core area in the development of mathematical thinking because it helps build skills like visualization, deductive reasoning, orientation, and problem-solving. Geometry is the foundation for many advanced areas of mathematics like trigonometry, calculus, coordinate geometry, topology, and even computer graphics and robotics. Geometry can thus be said to be a bridge between concrete experience and mathematical structure.

Geometry is one of the first areas of mathematics that students are exposed to in primary education after arithmetic. Students are introduced to basic concepts of geometry like patterns, symmetry, and spatial awareness. Angles and measurement are introduced as concepts that help build formal reasoning. For instance, one of the first concepts introduced is the angle sum theorem of triangles:

$$\angle A + \angle B + \angle C = 180^\circ$$

These properties will become essential elements for further mathematical learning. Therefore, primary school teachers become the first agents of geometric knowledge, and their conceptual clarity and attitudes towards geometry become of great importance.

2. LITERATURE REVIEW

Sahinkaya and Kilic (2021) examined how pre-service primary school teachers understood geometric concepts using a variety of metaphorical terms, as well as their metaphorical interpretations of geometry. The results showed that participants frequently connected geometry to ideas like order, space, structure, and organization, which reflected both their emotional and cognitive reactions to the topic. These metaphors revealed teachers' perceptions of geometry's nature and function in education, emphasizing both its clarity and, in certain situations, its underlying difficulties. The study emphasized that metaphor analysis could be a useful tool in teacher education for obtaining deeper insights into teachers' thinking and enhancing geometry teaching practices. It concluded that metaphors served as significant indicators of teachers' conceptual understanding and attitudes toward geometry.

Akçay and Çil (2021) found that participants produced a wide range of metaphors representing both positive and negative opinions when the metaphorical perceptions of primary pre-service teachers regarding mathematics, the teaching process, and mathematics instruction were investigated. While some pre-service teachers saw mathematics as a helpful tool for problem-solving and practical applications, many others saw it as a challenging and complex subject. According to Their et al.'s study, these metaphors encapsulated instructors' opinions of instructional strategies, difficulties in learning mathematics, and the usefulness of the subject. Such metaphorical understandings were found to have an impact on how future educators viewed teaching mathematics and engaged with students. The study underlined how important it was to comprehend these ideas in order to enhance teacher education programs, since addressing negative beliefs and promoting positive viewpoints could result in more successful and interesting methods of teaching mathematics.

Taş (2016) examined pre-service teachers' metaphorical conceptions of math and math education in the context of preschool teacher training and discovered that participants had a variety of, frequently divergent, opinions. While some pre-service teachers saw mathematics as fun, relevant, and crucial for cognitive development, many others used metaphors related to dread, difficulty, and abstraction to characterize the subject, reflecting negative attitudes and anxiety. The results indicated that teachers' future instructional practices were significantly influenced by these metaphorical conceptions, especially with regard to how they introduced and clarified mathematical topics to younger students. The study highlighted that while positive metaphors could foster stimulating and encouraging learning settings, negative perceptions could impede successful teaching. In order to enhance both teaching efficacy and students' early encounters with mathematics, Taş emphasized the significance of cultivating a positive and constructive metaphorical framework during teacher education programs.

Gambini and Lénárt (2021) explored how in-service and pre-service math teachers' conceptualizations of fundamental geometric ideas changed with different degrees of teaching experience. The results showed that prior learning experiences and conceptual clarity had a significant impact on teachers' geometric thinking, with many participants exhibiting superficial knowledge despite experience. It was shown that both pre-service and in-service teachers frequently had trouble making meaningful connections between ideas and providing logical explanations of basic geometric concepts. The study concluded that in order to improve overall teaching and learning effectiveness in geometry, teacher education and professional development programs should prioritize strengthening conceptual understanding, reasoning skills, and reflective practices. This is because effective geometry teaching necessitates a deeper engagement with basic geometric thinking.

3. THEORETICAL FOUNDATIONS OF GEOMETRY

Geometry, as a central area of mathematics, is based on a well-structured theoretical basis that explains the nature of space, shape, and spatial relationships. The theoretical basis of geometry is concerned with the examination of basic concepts like points, lines, planes, angles, and solids, as well as the axioms and postulates that regulate them. Based on these basic ideas, geometric knowledge is developed using logical reasoning, deductions, and proofs. The above-mentioned principles are of utmost importance not only for the understanding of geometry as a mathematical area but also for the comprehension of how learners and prospective teachers conceptualize geometric notions. In relation to teacher education, these theoretical foundations form the basis for effective teaching, visualization, and the development of deeper spatial and analytical thinking.

3.1 Definition of Geometry

Definition:

Geometry is one of the basic branches of mathematics that involves the study of shape, size, position, and spatial relationships. Geometry is concerned with the knowledge of properties and measures of basic geometric objects like points, lines, surfaces, and solids. Geometry is the backbone of spatial reasoning and logical thinking, and it plays a vital role in mathematics as well as in various applications like architecture, engineering, design, astronomy, and navigation.

Mathematically, geometry can be defined as the study of a set of objects:

$$G = \{P, L, \Pi, S\}$$

Where,

- Pdenotes points, which represent exact locations in space (e.g., $P(x, y)$),
- Ldenotes lines, which extend infinitely in two directions (e.g., $y = mx + c$),
- Πdenotes planes, which are flat two-dimensional surfaces that contain infinitely many points and lines, and
- Sdenotes solids, which are three-dimensional figures such as cubes, spheres, and pyramids occupying volume.

Geometry is not only about representation but also about logic. Geometry is built upon some assumptions called axioms and postulates, from which some important theorems are deduced. For instance, the triangle angle sum theorem is one of the most important theorems in Euclidean geometry:

$$\angle A + \angle B + \angle C = 180^\circ$$

Thus, geometry forms a structured system for analyzing space, understanding patterns, and building mathematical knowledge through definitions, axioms, theorems, and deductive reasoning.

4. METAPHORIC PERCEPTIONS IN MATHEMATICS EDUCATION

4.1 Meaning of Metaphor in Learning

A metaphor is generally understood to be a significant cognitive and linguistic device that assists a person in comprehending complex notions by associating them with more familiar experiences. In mathematical learning, a metaphor enables a person to develop meaning by associating formal mathematical concepts with everyday objects, situations, or pictures. As geometry involves abstract spatial thinking, metaphoric thinking is particularly helpful in making geometric concepts more accessible and meaningful.

Definition:

A metaphor can be described as a conceptual mapping:

$$M: A \rightarrow B$$

where an abstract domain A(such as geometry) is interpreted through a concrete domain B(such as a road, puzzle, or building).

For example:

$$\text{Geometry} \approx \text{Blueprint of the World}$$

This metaphor illustrates the concept that geometry is a structural plan for understanding space and design. Therefore, metaphors create deeper meaning beyond formal definitions and support stronger conceptual understanding.

4.2 Importance of Metaphors for Teacher Development

Metaphors are very important in creating pre-service teachers' views of mathematics and their future teaching practices. The metaphors used by teachers in geometry influence the way they teach geometry in their classrooms. Positive metaphoric perceptions can inspire teachers to teach in an interesting and meaningful way, while negative metaphors can cause teachers to fear teaching geometry or teach it in a rigid manner.

Metaphors assist teachers to:

- Enhance better and more integrated conceptual understanding
- Overcome difficulties and anxieties associated with abstract thinking
- Emphasize creativity in presenting geometric concepts

- Relate geometry to real-life experiences
- Develop approaches that are more student-friendly for the teaching of shapes, space, and patterns

Thus, the investigation of metaphorical perceptions among pre-service primary school teachers can be highly informative in understanding their cognitive structures and can help in improving geometry education at the foundational level.

5. CORE EUCLIDEAN GEOMETRY CONCEPTS

To grasp the metaphoric views of geometry held by prospective primary school teachers, it is necessary to first introduce the formal definition of geometry as defined in the Euclidean tradition. Euclidean geometry, developed systematically in Euclid's Elements, is the basic framework of most geometric concepts taught in primary and secondary education. It is founded on simple undefined concepts such as points and lines, as well as axioms and postulates from which theorems are logically deduced.

The basic concepts of Euclidean geometry include the examination of fundamental shapes, angles, parallel and perpendicular lines, triangles, polygons, circles, and three-dimensional solids. These concepts serve as the foundation of geometry. For example, a point in a plane is defined by coordinates:

$$P(x, y)$$

and the distance between two points $P(x_1, y_1)$and $Q(x_2, y_2)$is given by the Euclidean distance formula:

$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Similarly, the equation of a straight line, one of the most fundamental geometric objects, may be written as:

$$y = mx + c$$

where $m$denotes the slope and $c$the intercept. The slope itself can be expressed as:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Circles also play a central role in Euclidean geometry, and their standard equation is:

$$x^2 + y^2 = r^2$$

where $r$represents the radius.

Fundamental results such as the triangle angle-sum property:

$$\angle A + \angle B + \angle C = 180^\circ$$

6. FUNDAMENTAL DEFINITIONS

In Euclidean geometry, it is necessary to have a clear understanding of basic geometric concepts because all other concepts are based on these basic ideas. The basic ideas of geometry are points, lines, and planes, which are the basic objects of geometry. These ideas are undefined, and they are defined in terms of their properties. Using these ideas, other complex geometric objects like angles, polygons, circles, and solids are defined. In analytic geometry, these basic objects can also be defined using algebraic equations. This enables geometry to be studied in two ways: visually and algebraically. For instance, the distance between two points $P(x_1, y_1)$and $Q(x_2, y_2)$is given by:

$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Similarly, geometric properties like slopes, intersections, and angles can also be defined mathematically, giving a more deductive framework to geometric ideas. The following definitions of point, line, plane, and angle are the basic building blocks of Euclidean geometry.

6.1 Point

A point has no dimension—only position.

Symbol:

$$P(x, y)$$

6.2 Line

A line is a set of infinitely many points extending in both directions.

Equation of a line:

$$y = mx + c$$

where

m= slope,

c= intercept.

6.3 Plane

A plane is a flat surface extending infinitely.

Plane equation:

$$Ax + By + Cz + D = 0$$

6.4 Angle

An angle is formed by two rays sharing a common endpoint.

Angle measure:

$$\theta = \cos^{-1}\left(\frac{\vec{a} \cdot \vec{b}}{|\ \vec{a}\ ||\ \vec{b}\ |}\right)$$

7. EUCLID'S AXIOMS AND POSTULATES

Euclidean geometry is essentially an axiomatic system, which means that it is constructed from a few fundamental principles called axioms and postulates. These are assumed to be true without any need for proof and are used as the basis for all other theorems in geometry. Euclid's book "Elements" is one of the earliest attempts at structuring geometry in a deductive manner, starting with undefined terms like "point" and "line," and then postulates that define these terms. The beauty of Euclidean geometry is that it is able to derive complex conclusions through logical reasoning from very simple principles.

Euclid's five major postulates that define plane geometry and the concept of space in the Euclidean model are as follows.

- **Euclid's Five Postulates**

Line Segment Postulate: A straight-line segment may be drawn joining any two points. Symbolically, for two distinct points $P$and Q, there exists a unique line:

$$\bar{PQ}$$

Extension Postulate: A finite straight line may be extended indefinitely in a straight direction. This reflects the infinite nature of a geometric line.

Circle Postulate: A circle may be drawn with any center and any radius.The equation of a circle with center at the origin is:

$$x^2 + y^2 = r^2$$

where $r$is the radius.

Right Angle Postulate: All right angles are equal to one another.This establishes the uniform measure of a right angle:

$$90^\circ$$

Parallel Postulate: Through a point outside a given line, exactly one line can be drawn parallel to the given line.

If a line is expressed as:

$$y = mx + c_1$$

then its unique parallel line through another point has the form:

$$y = mx + c_2$$

where both lines share the same slope $m$but have different intercepts.

These postulates constitute the theoretical foundation of classical geometry. Based on these postulates, there are major theorems, including the angle sum property of triangles, properties of parallel lines, and properties of polygons. It is essential to understand Euclid's axioms and postulates because they not only constitute the logical foundation of geometry but also affect how students and prospective teachers conceptualize geometric space and reasoning.

8. KEY THEOREMS WITH PROOFS

The power of Euclidean geometry is not only rooted in its fundamental definitions and postulates but also in its formulation of significant theorems that express essential properties of geometric figures and spatial configurations. Theorems are logical deductions attained through deductive reasoning from axioms and prior proven statements. They constitute the central framework of geometric knowledge and serve as a foundation for problem-solving related to angles, distances, areas, and geometric figures. For prospective primary school teachers, it is imperative to comprehend significant theorems and their proofs, which not only improves conceptual understanding but also facilitates effective teaching. The following theorems, including the angle-sum theorem for triangles and the Pythagorean theorem, are fundamental geometric results that demonstrate the deductive nature of mathematical reasoning.

8.1 Theorem 1: Angle Sum of a Triangle

Statement:

The sum of interior angles of a triangle is:

$$\angle A + \angle B + \angle C = 180^\circ$$

Proof

Let triangle ABC. Draw a line through Aparallel to BC.

Using alternate interior angles:

$$\angle B = \angle 1, \angle C = \angle 2$$

Since angles on a straight-line sum to 180°:

$$\angle 1 + \angle A + \angle 2 = 180^\circ$$

Thus:

$$\angle A + \angle B + \angle C = 180^\circ$$

Proven.

8.2 Theorem 2: Pythagoras Theorem

Statement:

In a right triangle:

$$a^2 + b^2 = c^2$$

Proof (Outline)

Construct squares on each side of a right triangle. The area relationship gives:

$$\text{Area}(a) + \text{Area}(b) = \text{Area}(c)$$

Thus:

$$a^2 + b^2 = c^2$$

Proven.

Example Problem

If a right triangle has legs 6and 8:

$$c^2 = 6^2 + 8^2 = 36 + 64 = 100$$
$$c = 10$$

9. CONCLUSION

Geometry represents one of the most crucial areas of mathematics, providing the basis for logical reasoning, deduction, and the systematic understanding of geometric shapes and space. In the case of prospective primary school teachers, geometry is more than a set of formulas, definitions, and theorems, representing an area that demands conceptual interpretation. This paper has theoretically examined geometry by combining the formal structure of geometry, as presented in Euclid's works, with the cognitive function of metaphorical perception in mathematics education. Through the examination of basic geometric concepts such as points, lines, planes, angles, and solids, as well as Euclid's axioms and postulates, this paper emphasizes the deductive system on which geometry is based, while basic theorems such as the triangle angle-sum theorem and the Pythagorean theorem illustrate how geometric knowledge is logically developed into a proven truth.

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